SANDIA REPORT

SAND2004-4218
Unlimited Release
Printed August 2004

Approach and Development Strategy for an Agent-Based Model of Economic Confidence

James A. Sprigg, Jr., Craig R. Jorgensen, and Richard J. Pryor

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia is a multiprogram laboratory operated by Sandia Corporation,
a Lockheed Martin Company, for the United States Department of
Energy under Contract DE-AC04-94AL85000

Sandia National Laboratories

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

Printed in the United States of America. This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from
U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831

Telephone: (865)576-8401
Facsimile: (865)576-5782
E-Mail: reports@adonis.osti.gov
Online ordering: http://infoserve.sandia.gov/services/acq.html

Available to the public from
U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Rd
Springfield, VA 22161

Telephone: (800)553-6847
Facsimile: (703)605-6900
E-Mail: orders@ntis.fedworld.gov
Online order: http://www.ntis.gov/help/ordermethods.asp?loc=7-4-0#online

SAND2004-4218
Unlimited Release
Printed August 2004

Approach and Development Strategy for an Agent-Based Model of Economic Confidence

James A. Sprigg Jr.
Craig R. Jorgensen
Richard J. Pryor

Evolutionary Computing and Agent-Based Modeling
Sandia National Laboratories
P. O. Box 5800
Albuquerque, NM 87185-1110

Abstract

We are extending the existing features of Aspen, a powerful economic modeling tool, and introducing new features to simulate the role of confidence in economic activity. The new model is built from a collection of autonomous agents that represent households, firms, and other relevant entities like financial exchanges and governmental authorities. We simultaneously model several interrelated markets, including those for labor, products, stocks, and bonds. We also model economic trade-offs, such as decisions of households and firms regarding spending, savings, and investment. In this paper, we review some of the basic principles and model components and describe our approach and development strategy for emulating consumer, investor, and business confidence. The model of confidence is explored within the context of economic disruptions, such as those resulting from disasters or terrorist events.

Intentionally Left Blank

Contents

Figures

Tables

Approach and Development Strategy for an Agent-Based Model of Economic Confidence

Introduction

This paper explores theory and methods for modeling the economic activities and confidence of consumers, investors, and businesses. The economic activities include labor, production, capital expenditure, consumption, savings, and private investment. In reality, these activities occur within a complex system in which many economic actors behave and interact according to their internal objectives and constraints. Therefore, we present an agent-based approach to explore the complex decisions and interactions of autonomous economic participants, called agents, in such a system. The principal types of agents for this application are firms and households.

The theory and methods described herein are being applied in a new economic confidence model that Sandia National Laboratories is developing for the U.S. Department of Homeland Security (DHS). This model will allow DHS to analyze through simulation how terrorist acts and other types of economic disruptions affect our economy.

This paper is organized as a set of individually referenced topical sections. This first section, Introduction, describes the characteristics and benefits of agent-based models, explains the methods and assumptions underlying this approach, and gives a brief description of the development strategy we are following to build the new economic confidence model. The second section, Terror and Confidence, points out the various definitions that are used to describe economic confidence and addresses the relationship between terrorism and economic confidence. In the third section, Overview of Firms, we present our approach to implementing the firms, including their actions, their means of maximizing profit and determining production levels, and the planning algorithms that the firms employ in making decisions regarding prices and production levels. Examples are included to augment the discussion. In the fourth section, Overview of Households, we present our approach to implementing the households, including their actions; their means of consumption; their methods of borrowing, saving, and planning for the future; and the algorithms that the households employ in making decisions regarding consumption and saving. As with firms in the previous section, examples are provided to augment the discussion. The fifth and last section, Price Formation in the Stock Market, briefly touches on how investors participate in the stock market.

Background

Agent-based modeling generally refers to a computational approach that uses many independently functioning agents. Each agent is created from a software design and instantiated on a computer network, wherein the agents process information and pass messages. Each agent has access to certain information and the ability to communicate with other agents. Agents conduct their activities iteratively or continuously until the simulation halts. Generally, each agent is allowed to formulate a unique set of information upon which to act, thus allowing the agents to become unique. Additionally, agents are often designed with evolutionary algorithms, which allow the agents to learn about their environment and formulate unique sets of decision rules. Agent-based models are often used to observe aggregate activity for a population of agents, to observe how agents' decision rules can change as they adapt to their environment, and to determine distinct sets of equilibriums or *end states* for the model.

We expand upon a growing collection of Aspen[1] agent-based models that are used for the purpose of *economic simulation*. These models use classes of agents that simulate economic entities such as firms, banks, households, exchanges, and governments. For example, households might supply labor to firms in exchange for a wage. Firms transform labor into products, which are then sold to households for a price. Households and firms interact by passing messages, such as job postings, job applications, paychecks, and purchases. Similar activities are simulated for banks, governments, and other economic agents.

New Features

We extend existing agent-based models by adding features like a stock market and long-term financial plans to model consumer, investor, and business confidence. We pursue these extensions by incorporating the prerequisite principle of *intertemporal substitution*. This principle refers to a trade-off between having something now and having it later. For example, whenever a household saves money for a future vacation, it substitutes intertemporally (across time) because it is choosing to spend (consume) less now so that it can consume more in the future. Similarly, whenever a household borrows a car loan, it substitutes intertemporally because it obtains use of the car now, though it will give up some of its disposable income in the future until the loan is paid off.

Consumer confidence is specific to household agents and refers to households' confidence in their future income streams. If households become concerned that their future income streams are in jeopardy (by risk of wage cuts or layoffs), they might also worry that they will not be able to support their planned spending patterns.[2] This might lead many households to spend less and save more to lessen the potential impact if future income does fall. Spending less and saving more is a form of intertemporal substitution in which a household reallocates current consumption[3] to the future. This action unfortunately exacerbates economic slowdowns by

[1] Developed by Sandia National Laboratories by Richard Pryor, Dianne Barton, David Schoenwald, et al.

[2] Factors not related to employment and income that might affect consumer confidence are beyond the scope of this discussion.

[3] *Consumption* refers to the use of *final goods and services* purchased by households for their own use. Consumption excludes household purchases made for the purpose of producing and selling goods and services. For

reducing sales to consumers, which reduces the revenues of firms and creates pressure on firms to cut wages and lay off employees.

Investor confidence [in our model] is also specific to household agents. Households invest in stocks for the uncertain prospect of earning capital gains, a form of future income, for the sake of future consumption.[4] If households become concerned that their future wealth and income streams are in jeopardy as a result of falling stock prices and dividends, they might also worry that they will not be able to support their planned spending patterns. This might lead many households to convert stock holdings into less risky assets such as bonds or cash savings. Unfortunately, this action exacerbates the problem by driving stock prices down and reducing the capitalization of firms.

Business confidence is specific to firm agents and refers to firms' confidence in their profit streams and in their ability to raise capital via the stock market. If firms become concerned that their future profit streams are in jeopardy (by risk of declining consumer demand), they might also worry that they will not be able to support planned capital and payroll expenditures. This might lead many firms to cut costs, reduce planned capital expenditures, freeze hiring, cut wages or lay off employees in order to reduce the impact of potential revenue shortfalls and maintain profitability. These actions unfortunately exacerbate the problem by reducing household income and convincing households to reallocate more income into savings. Stock markets magnify the issue by increasing the pressure on firms to demonstrate short-term profitability in order to satisfy investors.

We introduce the *stock market* to allow households to trade current consumption for stock in firms. Households do so with the expectation of earning capital gains, which translate into future consumption. This market also allows firms to exchange stock in their business for working capital. A firm that participates in the stock market must account for the prospect that poor financial performance can impact investors' relative confidence in the firm and reduce the firm's level of capitalization, thereby hindering the firm's ability to plan. Similarly, a household that buys stock must account for the prospect that stock prices can reduce the household's wealth, thereby limiting its ability to plan. The stock market, combined with banking and bonds markets, will provide a reasonably comprehensive model of financial markets.

---

example, a household's purchases of office products for a home business are not included as household consumption; such items are considered *intermediate goods* used for the creation of additional goods and services. As demonstrated above, goods and services are classified as *intermediate* or *final* based on their intended use. Intermediate goods are those used to produce other goods for sale to firms or households, whereas final goods are those sold strictly to households and strictly for "personal" use. As a rule-of-thumb, *final* goods and services are those purchases by households that are classified as personal expenses, rather than as business expenses, for tax purposes.

[4] We note that many firms are capitalized primarily by institutional investors rather than by individual investors. However, distinctions between the investment behavior when funds are controlled by institutional traders rather than by individual investors are beyond the scope of this paper. We simply assert that such institutions derive their funds in some fashion from households, and that households' incentives are key to aggregate investment behavior.

Why Agent-Based Models?

Agent-based models offer a number of benefits in the analysis of complex systems:

First, from a practical perspective, agent-based models provide a distinct platform for operational analysis because the agents and their decisions and interactions can be described intuitively rather than abstractly, as is the case for sophisticated mathematics and subtle assumptions.

Second, agent-based models are well suited for a population of decision makers that have a diverse set of characteristics, as compared with analytical methods that often require restrictive simplifying assumptions.

Third, agent-based models allow us to model complex sets of relationships that cannot be explicitly modeled using statistics-based econometrics due to issues of data availability, degrees of freedom, or collinearity. And even when statistical methods provide predictive models, these methods generally do not allow the analyst to explore many underlying relationships[5] (whether or not such underlying relationships are even explicitly represented in the statistical model). In contrast, simulations can measure and record, for analysis, *all* data that are explicit to the underlying processes in the simulations. Further, econometric methods usually estimate empirical relationships according to an observed data range, but such estimates often break down when extrapolating outside of that range.[6] In contrast, one can verify agent simulations from the ground up according to the underlying distribution of input parameters and then run the simulation for scenarios that have never occurred.

A fourth benefit of agent-based models relates to the quantification of variables. Conventional comparative-static methods generally assume that an economic system is initially in some sort of equilibrium state. These methods analyze how changes in certain variables will move the system toward a new equilibrium state. Sometimes, however, the changes are derived without quantifying the equilibrium states. In contrast, simulations of agent-based models allow us to quantify all variables so that the observer can compare each variable before and after an event and observe the path of each variable to its new equilibrium, no matter how complex the system.

Fifth, agent-based models can provide a unique opportunity to study behavior over time. We incorporate evolutionary learning algorithms, such as genetic programs, into the agents. Learning algorithms, in general, have many applications and serve many purposes. In microeconomic simulation, they are particularly advantageous in allowing the analyst to relax assumptions regarding decision-making processes. For example, more conventional economic models typically assume that economic decision makers optimize according to the first- and second-order conditions of a fixed, twice-differentiable utility function. Such models are often powerful and useful for the study of underlying economic forces (under the assumption that economic

[5] Parametric statistical models rarely allow for sound investigation of high-dimensional n^{th}-order relationships. Rather, such models are usually limited to a few first- or second-order relationships.

[6] Statistically estimated relationships are local projections of more complex relationships. This sort of projection does not allow for backward projection outside the “relevant” range.

decision makers have perfect information), but they are not particularly useful for the study of behavior when decision makers know or understand little about their environment or their internal motivations. Agent modeling, on the other hand, employs learning algorithms to study how economic decision makers *learn to respond* to various choices under conditions of limited information and understanding.

Methods and Assumptions

The agent-based approach is unlike the traditional econometric approach in which empirical data are fitted to parametric models of measurable variables. Agent-based methods are more akin to network flow methods wherein the modeler designs the components of a system, attributes those components with certain functionality, assigns initial and boundary conditions, executes a search algorithm, and examines the final state of the model components. In the case of agent-based models, the modeler designs classes of agents to consider certain information, attributes those agents with certain functionality for sharing and processing information, instantiates populations of the agents, assigns initial and boundary conditions, executes the simulation for a duration of time periods, and examines the data generated by the activity of individual agents within the various agent classes. We therefore require substantial understanding about the underlying information, decisions, and transactions that are pertinent to real-world economic participants.

We employ generally accepted assumptions of economic behavior, when appropriate, as demonstrated in our design of household agents. Household behavior takes center stage in the model of confidence. We use the term *household* in the classic economic sense to refer to a standard unit of labor and consumer behavior.[7]

[7] Individual choices are intertwined within the household but are more-or-less independent from other households. For example, two individuals who form a family generally make certain joint decisions and joint purchases. In such cases, it is difficult or impossible, even in a theoretical formulation, to distinguish ownership and use of common purchases and assets between the two individuals; it is also difficult or impossible to distinguish the relative influence of each individual's preferences toward a common decision. These sorts of difficulties are largely avoided by studying the household as a composite decision maker whose behavior is determined by a bundle of weighted preferences corresponding to the members of the household.

Household decisions will require sound models of utility[8] and financial planning to simulate how households balance between current and future levels of consumption and how they choose to save and invest along a time horizon. This paper addresses these requirements by drawing upon decades of empirically supported household theory. In accordance with theory, we assume that households will estimate their budget constraint based on their wealth and expected income stream. We assume that households will estimate their utility, which we assume is related to the present value of consumption across many time periods. We assume that households prefer current consumption to future consumption and that households experience decreasing utility from consumption within any single time period. We make these assumptions because they are consistent with conventional wisdom and general knowledge of economic behavior.

Conversely, we avoid many assumptions that are used strictly to simplify the mathematics of economic theory. For example, we do not assume that households have perfect information. We do not assume that the households know (or even try to estimate) the distributions of stochastic variables (if stochastic processes are in the model). We employ a conventional log-linear utility

[8] *Utility* is an abstract concept used to study choices. Introductory lectures often describe utility as a generic measure of value or benefit or "happiness" derived by households from various forms of consumption, which usually provides a sufficient working definition of what we mean by utility. However, this description can be misleading, particularly in a contemporary culture that increasingly understands and embraces psychological principles such as compulsive, addictive, and self-destructive behavior. Thus, the aforementioned descriptions of utility (i.e., value, benefit, happiness) provide an inadequate context for introducing the economic principle of *rationality* and *rational agents*. Problems often arise as follows. When the concepts of *utility* and *rationality* are introduced in the simplest and most direct terms (as is usually the case), they imply an assumption that "all people behave rationally in their own best interest," which fails the *prima facie* litmus test applied by most audiences and leads many non-economists to reject economic thought and models outright. We must be clear. Utility is a model of predictive behavior, but it does not necessarily define the motivation behind such behavior. For example, when someone chooses option A over option B, they do so because they were motivated by some combination of internal factors, e.g., self-interest, psychological compulsion, subconscious desire, greed, charity, anonymous sacrifice, or an indifferent flipping of a coin. We do not intend to imply that the individual is "better off" by choosing A according to any preconceived social notion, or even that the individual is introspectively satisfied with the decision. We only intend to imply that the factors that motivated the individual's decision are relevant to the decision that was made. For example, suppose we conduct two experiments in which an individual is internally motivated to choose option A over option B, and similarly motivated to choose option B over option C. We simply define *utility* to be an ordinal ranking such that the utility of option A is greater than the utility of option B for this individual, and that the utility of option B is greater than the utility of option C for this individual. The purpose for such an ordinal ranking is to define preferences, leading to predictive models of behavior. To continue, the individual above had two choices with two options each. We shall define an ordinal ranking called utility, U, to examine the choices. In the first choice, we found that the utility of option A, U(A), was greater than the utility of option B; that is, $U(A) \geq U(B)$. The second choice revealed that $U(B) \geq U(C)$. We define *preference* to conclude that A is *preferred* to B and that B is *preferred* to C, as was demonstrated by the individual in the two experimental choices above. However, the utility concept allows us to consider all three options together as a set of preferences: $U(A) \geq U(B) \geq U(C)$. The ordinal properties of utility lead us to predict that A is *preferred* to C, even though there was no experiment for this choice. That is, based on our utility model, we predict that given a choice of A or C, the individual will be motivated to choose A. Similarly, given a choice of A, B, or C, the individual will choose A. Our only assumption is that the agent is internally consistent (whatever the motivating factors), not that the agent conforms to some external notion of *rationality*. Note: This simple introduction does not address the many extensions to the application of utility theory, such as the notions that "people learn" or "people guess" or "people get confused" or "people change" or "people grow," and we do not intend to review a century's worth of utility theory here, although we do expand on some of these notions later as pertinent to this paper. This brief discussion was intended merely to clarify our use of the terms *utility*, *preference*, and *rationality* in the context of household choice.

function for each household, but we do not assume that any household can define its own utility function or its first-order optimality conditions.

In summary, agent-based microeconomic simulation can draw from many of the *results* from economic research without being *restricted* by some of the conventional research methods.

Development Strategy

For the new economic confidence model, we will design and build a software prototype using the C++ programming language. This prototype will provide an agent-based model in which C++ classes will represent economic agents. Upon execution of the program, the classes will be instantiated as objects, which will represent economic agents. The agents will perform functions, such as computation and message passing, in a sequence of time steps. Messages will be read from and written to a queue. Some agents will contain decision rules and learning algorithms that will determine their behavior.

We rely heavily on the object-oriented principle of inheritance. This approach allows us to introduce complexity in well-documented phases. For example, we used a *simple* household class to design and verify a simultaneous model of the labor and product markets with minimal complexity. We subsequently created a bonds household class to introduce into the model the principle of household finance via a bonds market. The bonds household class inherits the simple household class and includes only those methods and variables necessary for interacting in the bonds market.

The analyst will design simulations, i.e., scenarios, using a graphical user interface (GUI). The GUI will allow the analyst to specify certain global properties, such as the number of agents, as well as properties and initial conditions for the agents. The analyst will also use input files to specify the characteristics of agents, such as initial *amount of cash holdings* or *lists-of-employees*. We describe our approach for modeling firms and households in the following sections of this paper.

Aspen provides a versatile platform for development and modeling. However, we will design agents and algorithms for portability so that they may be modified and incorporated into other platforms.

Model Components and Complexity

We will develop an increasingly complex model of the macroeconomy as appropriate for modeling confidence and confidence-related activities. To do so, we will adopt various existing Aspen features as needed, such as taxation, unemployment programs, and bond and credit markets. We will also explore the impacts of macroeconomic characteristics such as aging, generation bulges, and interest rates.

In the case of complex decisions in simultaneous markets, we will identify joint equilibriums and evaluate the model based on its convergence to those equilibriums. We will explore the role of learning and compare the behavior of agents under different assumptions and algorithms.

Empirics and Validation

Empirical economic research utilizes statistical (econometric) methods to model relationships between measured economic variables. We intend to statistically model the variables in the simulation and compare our results with econometric findings.

References

Acemoglu, Daron, and Shimer, Robert. “Efficient Unemployment Insurance.” *Journal of Political Economy* 107 (October 1999): 893–928.

Arifovic, Jasmina. “The Behavior of the Exchange Rate in the Genetic Algorithm and Experimental Economies.” *Journal of Political Economy* 104 (June 1996): 510–41.

Basu, Nipa; Pryor, Richard J.; Quint, Tom. “ASPEN: A Microsimulation Model of the Economy.” *Computational Economics* 12 (December 1998): 223-241.

Bonabeau, Eric. “Agent-Based Modeling: Methods and Techniques for Simulating Human Systems.” *Proceedings of the National Academy of Sciences* 99 (May 2002): 7280–87.

Chen, Shu-Heng, and Yeh, Chia-Hsuan. “Toward a Computable Approach to the Efficient Market Hypothesis: An Application of Genetic Programming.” *Journal of Economic Dynamics and Control* 21 (1997): 1043–63.

Davidson, Carl; Martin, Lawrence; and Matusz, Steven. “The Structure of Simple General Equilibrium Models with Frictional Unemployment.” *Journal of Political Economy* 96 (December 1988): 1267–93.

Ehrlich, Isaac, and Lui, Francis T. “Social Security, the Family, and Economic Growth.” *Economic Inquiry* 36 (July 1998): 390–409.

Judd, Kenneth L. “Computational Economics and Economic Theory: Substitutes or Complements.” *Journal of Economic Dynamics and Control* 21 (1997): 907–42.

Kiyotaki, Nobuhiro, and Wright, Randall. “On Money as a Medium of Exchange.” *Journal of Political Economy* 97 (August 1989): 927–54.

Piggott, John, and Whalley, John. “The Tax Unit and Household Production.” *Journal of Political Economy* 104 (April 1996): 398–418.

Swanson, Charles E., and Kopecky, Kenneth J. “Lifespan and Output.” *Economic Inquiry* 37 (April 1999): 213–25.

Yildiz, Muhamet. “Waiting to Persuade.” *Quarterly Journal of Economics* 119 (February 2004): 223–48.

SAND Reports

Barton, Dianne C., and Pryor, Richard J. *An Evaluation of the Convergence Properties of a Parallel Genetic Programming Method*. SAND2002-3276. Albuquerque, N.M.: Sandia National Laboratories, October 2002.

Barton, Dianne C.; Eidson, Eric D.; Schoenwald, David A.; Cox, Roger G.; and Reinert, Rhonda K. *Simulating Economic Effects of Disruptions in the Telecommunications Infrastructures*. SAND2004-0101. Albuquerque, N.M.: Sandia National Laboratories, January 2004.

Barton, Dianne C.; Eidson, Eric D.; Schoenwald, David A.; Stamber, Kevin L.; and Reinert, Rhonda K. *Aspen-EE: An Agent-Based Model of Infrastructure Interdependency*. SAND2000-2925. Albuquerque, N.M.: Sandia National Laboratories, December 2000.

Basu, N., and Pryor, Richard J. *Growing a Market Economy*. SAND1997-2093. Albuquerque, N.M.: Sandia National Laboratories, September 1997.

Basu, N.; Pryor, Richard J.; Quint, T.; and Arnold, T. *Aspen: A Microsimulation Model of the Economy*. SAND1996-2459. Albuquerque, N.M.: Sandia National Laboratories, October 1996.

Pryor, Richard. J.; Marozas, Dianne C.; Allen, A.; Paananen, O.; Hiebert-Dodd, K.; and Reinert, R. K. *Modeling Requirements for Simulating the Effects of Extreme Acts of Terrorism: A White Paper*. SAND98-2289. Albuquerque, N.M.: Sandia National Laboratories, October 1998.

Pryor, Richard J.; Basu, N.; and Quint, T. *Development of Aspen: A Microanalytic Simulation Model of the U.S. Economy*. SAND1996-0434. Albuquerque, N.M.: Sandia National Laboratories, February 1996.

Slepoy, Natasha A., and Pryor, Richard J. *Analysis of Price Equilibriums in the Aspen Economic Model under Various Purchasing Methods*. SAND2002-3693. Albuquerque, N.M.: Sandia National Laboratories, November 2002.

Terror and Confidence

Economic confidence is an abstraction without strict definition. Some define confidence in terms of indexes calculated from survey data. Others refer to confidence as an underlying immeasurable determinant of producer and consumer demand. However, any definition of confidence must surely relate to expectations and perception of risks and uncertainties. For example, consumer confidence is derived from, or defined by, a household's expectations for financial stability, risk, and opportunities. Investor confidence is based on many factors, including economic stability and growth. Business confidence is based on expectations for market demand. New risks or increased uncertainties will impact expectations and cause various households and firms to react in various ways, depending in part upon their assets and wealth, time horizon, and risk preferences. Thus, the discussion of confidence ultimately revolves around a discussion of economic and financial expectations. New information regarding economic and financial variables will therefore influence confidence.

In a general sense, we would expect confidence to be detrimentally impacted by new information in two ways. First, we would expect most households and firms to downgrade their expectations in response to new information of previously unaccounted economic or financial risks. Second, we would expect risk-averse households and firms to downgrade their expectations in response to new uncertainties regarding economic and financial conditions.

The relationship between terrorism and economic confidence is unclear. Some suggest that confidence was reasonably robust in the wake of 9/11 (Garner 2002; Shapiro 2003). Others suggest that increased uncertainties can reduce confidence and substantively harm the economy (Desroches and Gosselin 2002).

The question of confidence is part of a broader set of questions regarding the economic impact of terrorism. The DHS (U.S. Department of Homeland Security) is pursuing answers to such questions through various means, such as through the Homeland Security Center for Risk and Economic Analysis of Terrorist Events, as well as through other portfolios. There are many methods and data sources for analyzing questions of security and risk, but many new computational capabilities will only enhance our nation's ability to define and address the relevant questions. We anticipate that agent-based modeling will provide a useful framework for both (1) modeling economic decisions and (2) capturing information from data sources.

References

Baumol, William J. "Long Run Consequences of the Terrorist Acts – Possible Stimulating Effects." *Atlantic Economic Journal* 26 (December 2001): 358–59.

Blomberg, S. Brock; Hess, Gregory D.; and Weerapana, Akila. "Economic Conditions and Terrorism." *European Journal of Political Economy* 20, no. 2 (June 2004): 463–78.

Desroches, Brigitte, and Gosselin, Marc-Andre. "The Usefulness of Consumer Confidence Indexes in the United States." Working Paper no. 2002-22. Ottawa: Bank of Canada, August 2002.

Epstein, Joshua M. “Modeling Civil Violence: An Agent-Based Computational Approach.” *Proceedings of the National Academy of Sciences* 99 (May 2002): 7243–50.

Garner, C. Alan. “Consumer Confidence After September 11.” *Federal Reserve Bank of Kansas City Economic Review*, no. QII (2002): 1–21.

Ito, Harumi, and Lee, Darin. “Assessing the Impact of the September 11 Terrorist Attacks on U.S. Airline Demand.” Working Paper no. 2003-16. Brown University and LECG Corporation, 2003.

Knickerbocker, Brad. “Mad Cow Effects Ripple through Food Economy.” *The Christian Science Monitor*, 12 March 2004.

Lenain, Patrick; Bonturi, Marcos; and Koen, Vincent. “The Economic Consequences of Terrorism.” Economics Department Working Paper no. 334. Organization for Economic Co-operation and Development (OECD), July 2002.

Lugar, Richard. “Beating Terror.” *Washington Post*, 27 January 2003.

Rothfeder, Jeffrey. “Terror Games.” *Popular Science* 264, no. 3 (March 2004).

Shapiro, Robert. “Al-Qaida and the GDP: How Much Would Terrorism Damage the U.S. Economy? Less Than You’d Expect.” *The Dismal Science*, 28 February 2003. Available at http://www.slate.msn.com/id/2079298/ (accessed 09 July 2004).

Smith, Roger. “Counter-Terrorism Modeling and Simulation: A New Type of Decision Support Tool.” *Signal Magazine*, December 2001.

Virgo, John M. “Economic Impact of the Terrorist Attacks of September 11, 2001.” *Atlantic Economic Journal* 26 (December 2001): 353–57.

Overview of Firms

The basic purpose of a firm in our model is to produce and sell products in the marketplace in order to earn profits. Firm-based production has become the societal norm in most cultures because firms can generally acquire labor and capital in amounts and proportions that make them more productive (efficient) than alternative approaches to production. This is so much the case that one can scarcely imagine a household that grows its own food, weaves its own cloth, and provides its own entertainment. Productive efficiency creates higher per-capita product, which can be distributed among the workers, owners of capital, and consumers. In a money economy, these benefits are distributed in the form of higher wages, higher profits, and lower prices.

We will design a simulation in which the firms learn to plan production levels via decision algorithms to optimize some measure of profit. This approach requires that the firms search for optimal decision rules for the set of possible economic states. The simulation framework requires that each firm continually recalculate its profit (fitness) to determine whether its decisions have made it better off.

In general, firms purchase input factors, produce and sell products, and attempt to do so in a fashion that maximizes profit. We will design a C++ class to handle the data structures and methods that are required by firms. Examples of data structures might include a list of employees or an array of consumer-request messages. An example of a method might include a production function that translates input factors into products for sale in the market.

Key aspects of our implementation of firms include the following:

- **Inventory and payroll.** Each firm will establish an inventory record, which will be credited whenever the firm produces new quantities of the corresponding product and debited whenever the firm sells the product in the marketplace. Each firm will also establish a payroll record for each employee to record the employee’s wage and ID number. Records will be added and deleted each time an employee is hired, fired, or quits.

- **Profit and accounting.** Each firm will maintain business records, such as sales price, cost, revenue, profit, and excess demand, which will be updated with each new series of transactions.

- **Choices and decisions.** Each firm will seek to optimize some set of objectives. The firm’s objectives will generally be defined by some measure of financial fitness such as profit. All activities and decisions of the firm will relate to such fitness measures. The foremost decision will be to determine production levels. Related to that decision will be decisions regarding employment and pricing.

- **Message passing.** Each firm will read and send messages for various purposes. Some messages, e.g., job advertisements, will be posted publicly for general access. Other messages, such as job offers, will be sent privately to a specific identified agent. Similarly, some of the messages that a firm reads might be public postings like an advertisement of intermediated goods available for purchase, or private messages like a job application.

Profit Maximization

The classic *theory of the firm* assumes that firms optimize their profit function under a cost constraint. We consider the simplest example of such optimization. A firm pays wage "w" to employees for "L" hours of labor at a total cost of $C = w \cdot L$. The firm produces "q" units of product as a function of labor: $q = q(L)$. The firm sells its product at price "p" for a total revenue of $R = p \cdot q$. If we suppose for this example that wages and price are determined exogenously, then profit "π" is given by $\pi = R - C = p \cdot q(L) - w \cdot L$. The firm will maximize profit under the following first-order condition:

$$\partial\pi/\partial L = p \cdot \partial q/\partial L - w = 0,$$

which implies

$$\partial q/\partial L = w/p.$$

The term "$\partial q/\partial L$" is called the *marginal product of labor* and depends on the technology used by the firm to transform labor into product. The properties of the function q(L) determine the profit choices facing the firm. For example, consider a simplistic case in which q(L) is linear: $q = k \cdot L$; in this case, the firm can only earn a positive profit if $k > w \cdot L/p$. Such a firm faces a simple choice: to produce or not produce based on wages and prices.

Economists incorporate concepts such as *economies of scale* and *diminishing returns* into this exercise by assuming that the production function can be approximated by a function that is twice differentiable with a positive first derivative and a negative second derivative. That is, production increases with labor, but at a decreasing rate. So the firm will only choose to engage in production if there is a range of production (i.e., a range of L and q) at which $\partial q/\partial L \geq w/p$. However, the firm is limited to how much it can produce profitably. Eventually, the firm reaches a critical quantity of labor L^* beyond which $\partial q/\partial L \leq w/p$. Thus, L^* represents the profit-maximizing *production level*.

These examples assume that prices and wages are fixed, and that the firm only decides whether to produce and how much to produce. There are industries where these conditions hold more-or-less, but firms in most industries usually have some control over the wages they pay and the prices they charge. Firms set wages to try to gain an advantage in the labor (input) market, and they set prices to try to gain an advantage in the product (output) market. We will discuss these more complicated sets of choices later, but first we explore the choice of production in the context of learning algorithms.

Cobweb Models

Cobweb[9] models are simple iterative exercises that examine how a firm chooses its production level when it is uncertain about the price it will obtain for its product. Cobweb models assume that there exists an equilibrium price and quantity that balances the supply and demand, but that the market begins out of equilibrium. That is, the production levels are initially

[9] The term "cobweb" arises because a cobweb model results in a series of price and quantity movements, which (when plotted on a standard graph of supply and demand curves) resemble a cobweb.

either too high (which implies a surplus of products), or the production levels are too low (which implies a shortage of products).

As we saw in our previous examples from our discussion of profit maximization, firms in a competitive market optimize by setting their production levels with respect to the market price for their product. Suppose, for example, that production levels are initially too low in the first period. In a competitive market, the market price will be relatively high (compared to the equilibrium price). The simple cobweb model supposes that firms will increase production levels for the second period based upon the currently high price. However, as firms introduce more units of product into a competitive market, the market price is driven below the equilibrium price. If the firms plan third-period production levels based on the low price in the second period, then they will underproduce, which will return the market once again to an excessively high price. As we iterate through time, we see a series of overcompensations by which the firms oscillate between overproduction and underproduction.

Ezekiel (1938) posed the cobweb theorem to explore whether price and production, if disturbed from their equilibrium, tend to gravitate back toward that normal. Cobweb models are used to explore the conditions under which the market (1) converges to equilibrium, (2) diverges away from equilibrium, or (3) oscillates. The outcome depends on the structure of the supply and demand schedules, as well as the method used by the firms to forecast prices. These models have more modern implications for the study of *rational expectations* and *monetary policy*.

For our purposes, cobweb models demonstrate the importance of expectations and forecasting in the definition of firm agents for an agent-based economic simulation because these models allow us to validate the performance of the simulation under certain definable test conditions.

Learning Algorithms

Arifovic (1994) summarizes some algorithmic extensions to a cobweb model. In the simplest algorithm, the firm just uses the current market price as its expectation for the next period's price. Ezekiel (1938) found under this scenario that prices and production will converge to equilibrium when the elasticity of supply and demand satisfies certain conditions (this is known as the *stable* case), but that prices and production will diverge when those conditions do not hold (this is known as the *unstable* case). In slightly more sophisticated algorithms, the firm might use past prices to forecast future prices based upon a sample-average or least-squares criterion. We appreciate all of these approaches and will likely use such methods for the purpose of testing. However, these approaches all lead to aggregate behavior that is artifactual of the rules imposed by the algorithm; that is, they do not allow the firms to be creative in learning how to plan production.

Arifovic presents a genetic algorithm by which firms can learn to set production. Each firm selects a binary string from a set of strings that correspond to incremental levels of production. Once a string is chosen, it is assigned a fitness based on the subsequent observed profit. Arifovic compared the genetic algorithm to the aforementioned forecasting approaches; she found that the genetic algorithm performed more desirably based on two criteria.

First, the genetic algorithm converged to equilibrium for both the stable case and the unstable case, whereas the sample-average approach failed to converge for the unstable case. We assert that convergence under normal conditions is a desirable result because we do not find empirical or experimental evidence of market prices that diverge in an oscillatory fashion based on increasing overcompensation by firms.

Second, the genetic algorithm exhibited price fluctuations prior to convergence, and the variance of those fluctuations was greater for the unstable case than for the stable case. In contrast, the variance of price fluctuations for the least-squares approach (which converged for both stable and unstable cases) did not differ. We assert that different variances for the stable and unstable cases is a desirable result because the firms have more to learn in the unstable case; thus, it is not unreasonable to expect that prices and production would fluctuate more for the unstable case on their path to equilibrium.

Production Planning

In reality, production planning is usually more complicated than simply selecting the number of units to produce. Below, we explore a few of the most common issues facing firms as they determine production levels.

We will define *firm* agents that determine their own production rate and store their output in an inventory. The firms will fulfill orders from customers out of the existing inventory. Using one or more genetic algorithms or similar methods, the firms will try to determine the production rate that maximizes profit subject to a series of cost constraints. Fitness for these algorithms will be defined by profit.

Capacity, Utilization, and Fixed Versus Variable Expenditures

The *scale* of a firm's operation usually refers to its production level and to the bundle of resources and assets, such as capital, labor, resources, and management, by which the firm operates. Economists split these resources and assets into two general categories: fixed costs, which determine capacity, and variable costs, which determine utilization. An increase in *fixed* (long-term) capital expenditures, such as facilities, vehicles, or machinery, will subsequently lead to an increase in capacity. For a given capacity, firms may purchase *variable* (short-term) inputs, such as fuel and energy, contract labor, office supplies, or transportation services, that will determine the level of *utilization* of the existing capacity.

An increase in capacity will clearly be accompanied by an increase in fixed costs. However, an increase in capacity will usually reduce the variable costs of production by allowing the variable inputs to combine more efficiently. Firms seek the most profitable balance of fixed and variable costs.

Firms can face various *scaling costs* when they increase their scale, such as costs pertaining to recruiting and training, capital acquisition, and process re-engineering. Similarly, firms can face costs when they decrease their scale, such as costs associated with severance packages or capital write-offs. Thus, depending on the nature of a firm's industry, a firm will often seek to maintain relatively steady levels of employment and capital instead of responding to every fluctuation in demand and market price.

There are no precise criteria for distinguishing fixed costs from variable costs. Such classifications depend on the life cycle of the input, the scaling costs, and the time frame being considered. However, general distinctions for broad classification of expenditures are useful for financial planning.

In this paper, we refer to the *scale of production* as a blend of both fixed and variable expenditures, without distinguishing between the two.

Inventories and Production Rates

In product markets, firms often store inventories of their product to maintain steady rates of production in the face of fluctuations in demand. Such firms try to plan their production rate so that inventories are large enough to satisfy surges in demand but small enough to keep inventory costs reasonably low.

Firms that maintain inventories expect those inventories to fluctuate within a planned range. However, if the demand for a firm's goods changes drastically, or for a sustained period of time, inventories will grow or decline outside of the planned range. This indicates to the firm that current production rates might no longer be appropriate for market demand, and can lead the firm to either increase its scale (if inventories are falling) or reduce its scale (if inventories are rising).

Firms plan production levels based on expected profitability, which is determined in part by input and output prices. In some markets, such as those for crude oil and other commodities, firms must plan production far in advance, before input and output prices are known. Thus, firms can suffer losses stemming from unfavorable price changes that occur after production has been planned. When inventory costs are low, firms might respond to such price changes by allowing inventories to grow until demand increases. However, inventory costs in some industries can be prohibitively high. Many such industries (those with long planning horizons and high inventory costs) have futures markets, and firms in these industries can hedge against unfavorable price changes by buying or selling futures contracts for pending deliveries months before the commodity is actually delivered.

Budget Constraint

In our model, firms will obtain working funds from three sources. The firms will (1) collect revenue from the sales of products, (2) borrow funds in the credit markets at an interest rate, and (3) issue stock to raise capital.

Financial Planning

Firms address their objectives over a multiple-period time horizon by forecasting demand for their products. From these forecasts, they devise business plans, which specify outlays of fixed capital (i.e., fixed-capital investments), and planned levels of payroll, purchases, and revenues. The business plan describes a multiple-period estimate of profits over some specified time horizon to demonstrate that the present value of future profits will exceed the costs of the initial capital outlay for the venture. The business plan also provides a discounted cash-flow analysis, which considers initial capital outlays and computes the discounted net present value of the proposed venture.

In a simple model, the present value of profit (π) from a future time period is discounted by the interest rate (r) as follows:

$$PV(\pi_t) = \pi_t/(1 + r)^{t-1},$$

where π_t is the amount of profit that will be earned "t" periods in the future. For example, suppose that π_t = \$100 is a future profit that will be earned in six months, and r = 2% is a monthly interest rate. We have t = 6 to represent six monthly time periods until the future profit is realized. Thus, the present value of the future profit is

$$PV(\pi_6) = \$100/(1.02)^6 = \$88.80.$$

The present value of the profit stream, which is the set of future profits ($\pi_{\{t\}}$), is calculated by summing across all such future time periods:

$$PV(\pi_{\{t\}}) = \sum \pi_t/(1 + r)^{t-1}.$$

In our simulation, each firm corresponds to a single venture involving the production and sale of its product. Thus, the firm must simply forecast its costs and revenues to estimate the present value of its expected profit stream. Of course, the accuracy of forecasts of costs will depend in part upon the accuracy of forecasting wages and input prices. Similarly, the accuracy of forecasting revenues will depend on the accuracy of forecasting product prices. Unexpected changes in wages and prices can increase or decrease profits from their expected levels. Thus, firms must constantly re-evaluate their production plans based upon changing prices.

Planning Algorithms

We will employ various types of algorithms at various stages of development to support the following decision process. In each planning phase, the firm computes its current fitness (derived from profit-related variables), determines its current state of being, and specifies the values for choice variables like price and production level. The second variable, production level[10], will be translated into more direct variables, such as number of employees, wage rate, or capital expenditure.

Aspen provides a library of documented algorithms to be employed when appropriate. For example, Slepoy and Pryor (2002) showed that genetic algorithms provide firm agents with an effective pricing model that achieves Nash equilibrium. We will draw from and add to the collection of decision algorithms as needed.

[10] To clarify, we use the term *production level* to refer to the *level* of production in a single period, which might alternatively be called the *production rate*. That is, we use the terms *production level* and *production rate* interchangeably.

Fitness

Fitness will be defined by the present value of a forecasted profit stream:

$$PV(\pi_{\{t\}}) = \sum \pi_t/(1 + r)^{t-1}.$$

Of course, future profit is a function of several uncertain prices and wages: $\pi_t = \pi_t$ (product and input prices, wage rate, inventory charges). Let $p_t^e \equiv E[p_t]$ denote the expected price in future time "t". The expected future profit is $\pi_t^e = p_t^e \cdot Q - C(p_{Xt}^e, w_t^e, c_t^e)$, where p_{Xt}^e denotes the expected price of inputs, w_t^e denotes the expected wage rate, and c_t^e denotes the expected inventory cost per unit. The expected present value of the future profit stream is

$$PV(\pi^e_{\{t\}}) = \sum \pi_t^e/(1 + r^e)^{t-1},$$

which also depends on the forecasted interest rate r^e. Changes to expectation will directly change the discounted value of the expected profit stream.

State Variables

We address the uncertainties described above by introducing a set of state variables (Ψ) to represent exogenous variables in the utility function. We suggest a few specific state variables in this paper (see Example of an Assessment Step below), but the choice of state variables will likely be modified in the course of model development.

Pricing

We intend to design scenarios that will reasonably emulate competitive markets with many suppliers. Thus, firms have a vested interest in accurately forecasting the equilibrium price, p^*. If a firm sets its price above the market, then most or all consumers will purchase from other firms, leaving the overpriced firm with excess inventory and lower profits (possibly negative). If a firm set its price below the market, then it should get many buyers and is likely to sell out its inventory; however, the firm's profits will be lower than they could have been at market price. In the simplest pricing scenarios, we will require firms to forecast the market price, then set their own prices equal to the market price: $p_{firm} = p_{market} = p(\Psi)$.

Eventually, we could explore firms that take different strategies toward pricing, or we could explore firms that learn or converge upon various pricing strategies, but we will not address these possibilities here.

Production

A firm will seek to determine an optimal level of production based upon price forecasts, upon its current inventory of product (V_Q) and inputs (V_X), and upon its labor force (L):

$$Q = Q(p^*, V_Q, V_X, L, \Psi).$$

Hiring and Purchasing

A firm might also explicitly make decisions regarding the purchase of inputs or the hiring of labor. That is, a firm might determine its target labor (L) based on the same (or similar) variables used to determine its production level:

$$L = L(p^*, V_Q, V_X, L, \Psi).$$

Similarly, a firm might use those variables to determine the purchase of nonlabor inputs (X):

$$X = L(p^*, V_Q, V_X, L, \Psi).$$

Remarks

We have listed four types of choice variables: product price, production level, and input factors (labor and other). The following examples describe a highly tentative approach for allowing firms to specify these variables. It remains to be determined how to best design algorithms for selecting these variables.

Example of an Assessment Step

In any planning phase, a firm (given its current state) must plan for future demand, which involves price and quantity. Thus, the firm must determine whether to raise, lower, or maintain the current price, and whether to increase, decrease, or maintain current production levels.

Regardless of the type of algorithms employed, firms will (1) identify their current state and (2) respond accordingly. For example, consider three *direct state* variables corresponding to profit, as listed in Table 1.

Table 1. Definition of State Variables

ψ_1	= 1 = 0	if profit is increasing: $d\pi_t/dt > 0$ otherwise
ψ_2	= 1 = 0	if inventory is increasing: $dV_q/dt > 0$ otherwise
ψ_3	= 1 = 0	if average costs are increasing: $\{C/Q\}_t > \{C/Q\}_{t-1}$ otherwise

In this case, we have n = 3 state variables. For any such collection of n *Boolean* (true or false) state variables, there are $N = 2^n$ states. In this case, we have $N = 2^3 = 8$ states for period t, where the set of possible *direct states* in period t is denoted by $\Psi_t = \Psi\{\psi_{1t}, \psi_{2t}, \psi_{3t}\}$.

We might also want firms to respond with regard to state transitions. We introduce a new set of *Markov* states: $\Phi_t = \Phi\{\Psi_t, \Psi_{t-1}\}$. There are $M_t = N_t \cdot N_{t-1}$ possible Markov states (in our model, N and M are constant across time). For this example, we have the following N = 8 direct states:

Ψ	ψ_1	ψ_2	ψ_3
1	1	1	1
2	1	1	0
3	1	0	1
4	1	0	0
5	0	1	1
6	0	1	0
7	0	0	1
8	0	0	0

And we have the following M = 64 Markov states:

Φ_t	Ψ_t	Ψ_{t-1}
1	1	1
2	1	2
…	…	…
7	1	7
8	1	8
9	2	1
10	2	2
11	2	3
…	…	…
62	8	6
63	8	7
64	8	8

Example of a Decision Step

Suppose a firm is seeking to choose a price. We partition this choice into two components: (1) an integer component for *disposition* (σ_D) and (2) a floating-point component for *magnitude* (σ_M). Disposition denotes the direction of the action taken, and magnitude denotes the extent of the action taken. For example, the disposition of a firm with respect to price can fall into one of three discrete categories. The firm can increase the price, lower the price, or hold the price constant. These three options are represented in the following chart by three possible values of σ_D:

σ_D	$= 1$ $= -1$ $= 0$	raise: $P_{t+1} > P_t$ lower: $P_{t+1} < P_t$ neither

Once a firm has decided its disposition toward pricing, it must decide the corresponding magnitude for this action. For example, if a firm decides to increase the price, will it increase the price a little or a lot? Magnitude denotes the new price as a proportion of the current price: $\sigma_M = P_1/P_0 \in [0, \infty)$.

In principle, it seems reasonable for a firm to first ask, “Should I raise or lower a price?” and then to ask, “OK, how much?” In practice, the disposition component (σ_D) allows for a discrete set of options from which the firm can choose. Discrete (rather than continuous) choices are important features for practical implementation of the decision algorithm.

Disposition: For a given Markov state, say $\Phi_0 = 5$, a firm has three options:

Φ_0	Ψ_0	Ψ_{-1}	σ_D
5	1	5	1
5	1	5	0
5	1	5	–1

This policy table (above) contains quite a bit of information. It shows that the firm was previously in direct state 5 (since $\Psi_{-1} = 5$), which implies the previous period had profit decreasing ($\psi_1 = 0$), inventory increasing ($\psi_2 = 1$), and average costs increasing ($\psi_3 = 1$). The chart shows that the firm is currently in direct state 1, in which profit is now increasing ($\psi_1 = 1$).

To understand how the firm selects from the three options, we introduce a *strength* parameter: $\omega \in [0, 1]$. The strength of each option in the current state represents the likelihood of selecting that option. For this example, we would have initiated all options at the start of the simulation to have equal strength ($\omega = 0.33$). However, each time an option is selected throughout the simulation, ω is increased or decreased depending on whether the decision resulted in a higher fitness for the firm. Suppose our current options have the following strengths (shown in the fifth, i.e., ω, column):

Φ_0	Ψ_0	Ψ_{-1}	σ_D	ω	σ_M
5	1	5	1	0.3	1.4
5	1	5	0	0.5	
5	1	5	–1	0.2	0.4

Choices and Magnitude: The three rows in the chart above imply three prospects: (1) a 30% chance of increasing the price by 40%, (2) a 50% chance of holding the price constant, and (3) a 20% chance of decreasing the price by 60%.

Example of an Evaluation Step

Suppose that (in the previous decision step) the algorithm selected the first option (increase price by 40%). The firm then sets its price accordingly and proceeds through a series of execution steps in which the firm sells its product in the marketplace. When the firm gets to the next evaluation step, it will determine whether fitness has increased or decreased.

Suppose that fitness has increased. In this case, the firm concludes that it made the correct choice for the given situation and increases the parameter $\omega_{(\Phi=5,\ \sigma D=1)}$ by some proportion. This step will increase the strength of that option (price increase) and thereby increase the probability of selecting the same option the next time the firm finds itself in state $\Phi = 5$. If fitness had decreased, the firm would have decreased the strength of the *price increase* option.

Continuing with the supposition that fitness has increased, we want the firm to consider whether it adjusted the price by the proper proportion. Based on well-founded economic principle and our theoretical framework, the firm experiences declining quantity demanded (sales) as the price rises. This implies that increasing the price will only increase profits to a point, but eventually fitness will begin to decline as a result of declining sales. We incorporate this principle by allowing the firm to adjust σ_M whenever it adjusts ω. Thus, once the firm determines that price increases are beneficial, it increases both ω and σ_M. Suppose the firm repeatedly finds itself in state $\Phi = 5$ and increasingly selects the first option (price increase) until σ_M approaches *unity* and fitness starts to decline upon evaluation. In response to such declining fitness, the firm will then begin to reduce the proportion of price increases whenever the *price increase* option is chosen. By adjusting the magnitude parameter, we expect the firm to converge to an optimal proportion of price increases.[11]

Remarks

The learning algorithms proposed above describe a general approach, which we will likely modify and extend during implementation of actual simulations.

References

Arifovic, Jasmina. "Genetic Algorithm Learning and the Cobweb Model." *Journal of Economic Dynamics and Control* 18 (1994): 3–28.

Blanchard, Olivier Jean, and Kahn, Charles M. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica* 48 (July 1980): 1305–11.

Campbell, Carl M. III, and Kamlani, Kunal S. "The Reasons for Wage Rigidity: Evidence from a Survey of Firms." *Quarterly Journal of Economics* 112 (August 1997): 759–89.

Ezekiel, Mordecai. "The Cobweb Theorem." *Quarterly Journal of Economics* 52 (1938): 279.

Kydland, Finn E., and Prescott, Edward. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (November 1982): 1345–70.

McGrattan, Ellen R. "A Note on Computing Competitive Equilibria in Linear Models." *Journal of Economic Dynamics and Control* 18 (1994): 149–60.

Muth, John F. "Rational Expectations and the Theory of Price Movements." *Econometrica* 29 (July 1961): 315–35.

Nerlove, Marc. "Adaptive Expectations and Cobweb Phenomena." *Quarterly Journal of Economics* 72 (1958): 227–40.

Slepoy, Natasha A., and Pryor, Richard J. *Analysis of Price Equilibriums in the Aspen Economic Model under Various Purchasing Methods*. SAND2002-3693. Albuquerque, NM: Sandia National Laboratories, November 2002.

[11] Convergence of σ_M is only for those cases when the firm selects the correct option. In our discussion, the economic structure causes the price change to converge to $\sigma_M < 1$. Unfortunately, the algorithm described here will also cause the strength to converge to $\omega < 1$ even if it should not; that is, even if the *pricing* option always improves fitness and other options always reduce fitness. We will return to this problem during development.

Smith, Vernon L. “Theory, Experiment and Economics.” *Journal of Economic Perspectives* 3 (Winter 1989): 151–69.

Spencer, David E. “The Relative Stickiness of Wages and Prices.” *Economic Inquiry* 36 (January 1998): 120–37.

Welford, C. P. “A Laboratory Analysis of Price Dynamics and Expectations in the Cobweb Model.” Discussion Paper no. 89-15. Tuscon: University of Arizona, 1989.

Overview of Households

Possibly the most fundamental agent in any market-based economic simulation is the household. The preferences of the households determine work effort, production incentives, prices, and interest rates. Households define the ends of virtually all market-based activity: the consumption of final goods and services.[12]

Unlike firms, households do not usually arise out of the pursuit of economic efficiency; they are a more fundamental social grouping. The primary distinction between households and firms in our models lies in their economic motivations. We model firms as being motivated by some direct financial measure such as fitness. We model households as being motivated by some abstract concept of *utility*. However, household utility clearly relates to household finances because a necessary component of utility is the consumption of goods and services, which are generally purchased in markets. We, therefore, find similarities in the financial decision processes of households and firms.

We present a model of household financial planning based upon the conventional principles of utility theory and intertemporal substitution. We then present an evolutionary learning algorithm by which household agents can systematically learn to modify their savings and investment decisions in an optimal fashion. This method is a step toward a model of consumer confidence. Ultimately, we will define a method by which households continually allocate funds between three options: current consumption, [risk-free] savings, and [risky] stock investments.

In general, households earn income and consume final goods and services, and attempt to do so in a fashion that maximizes utility (see the discussion of utility under Financial Planning below). We will design a C++ class to handle the data structures and methods that are required by households. Examples of data structures might include a list of desired consumption goods that must be purchased, or an array of stocks held in an investment portfolio. An example of a method might include an income-allocation method that apportions current income among consumption, savings, and risky investments.

Key aspects of our implementation of households include the following:

<u>Labor Markets</u>: Households must usually earn income to purchase products. They earn income by entering the labor market, working for a firm, and earning a wage.

<u>Product Markets</u>: Households consume by purchasing products in the *final goods markets* (i.e., *consumer goods markets* or *retail markets*). Households consume bundles of products based

[12] *Consumption* refers to the use of *final goods and services* purchased by households for their own use. Consumption excludes household purchases made for the purpose of producing and selling goods and services. For example, a household's purchases of office products for a home business are not included as household consumption; such items are considered *intermediate goods* used for the creation of additional goods and services. As demonstrated above, goods and services are classified as *intermediate* or *final* based on their intended use. Intermediate goods are those used to produce other goods for sale to firms or households, whereas final goods are those sold strictly to households and strictly for "personal" use. As a rule-of-thumb, *final* goods and services are those purchases by households classified as personal expenses, rather than as business expenses, for tax purposes.

on their internal preferences and the relative product prices, and they substitute between products based on the relative price changes.

Financial Markets: Households can earn income in the form of capital gains by investing in the stock market. Capital gains represent a return for risk taking and patience; the household foregoes consumption in one time period for the prospect of earning a return in a later period.

Savings and Net Worth: Households generally save portions of their income to handle spikes in consumption or to maintain consumption levels during possible periods of unemployment. Households generally increase their net worth through savings during earlier phases of their life cycle, then consume their new worth during retirement.

Consumption and Life-Cycle Utility: Households will seek to optimize some set of objectives. A household's objectives will generally be defined by some measure of fitness that itself is defined by discounted utility derived from consumption.[13] All household activity and decisions will center on that fitness measure. The foremost decision will be to determine consumption levels. Related to that decision will be decisions regarding savings and investment.

Work Effort

Before examining how households spend their income, we first examine a more basic trade-off: the trade-off between leisure and work effort. We treat leisure as a form of consumption for the household; that is, more leisure makes the household better off. However, spending time at work rather than at leisure generates income, which allows the household to consume goods and services. Therefore, households choose between leisure and consumption.

Here, we present the simplest of representations of this relationship with a one-period model. We adopt the convention that utility (U) is monotonic with respect to each of two factors, consumption expenditure (C) and leisure (L). We use the standard Cobb-Douglas utility function

$$U = [L^{\alpha} \cdot C^{\beta}]^{\lambda},$$

where α and $\beta \in [0, 1]$. This simple model is graphed in Figure 1, wherein we see that utility increases with respect to both leisure and consumption.

[13] Eventually, we might incorporate the concept of leisure as a form of consumption. For the foreseeable future, however, we will implicitly address this principle by defining utility as an exponential function of consumption with homogeneity of degree less than one. This cavalier treatment of leisure will be corrected later.

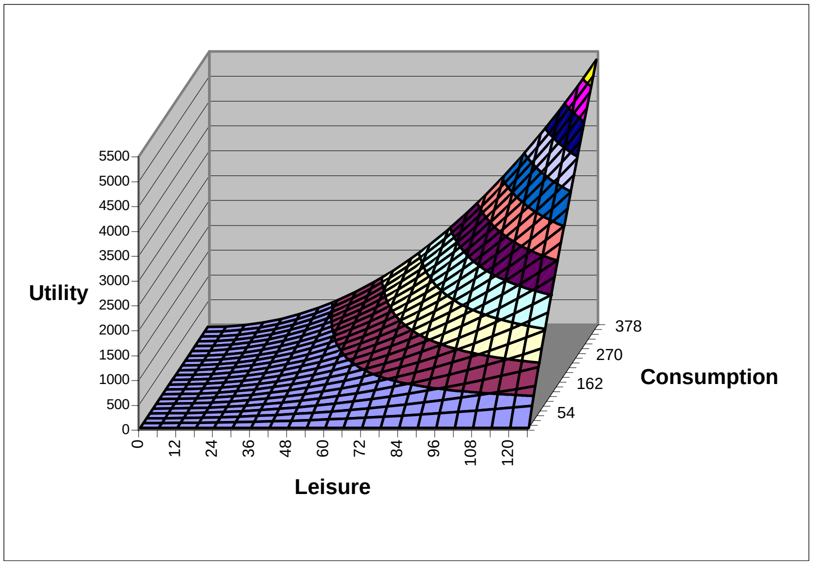

Figure 1. Household utility as a function of leisure and consumption.

Elasticity

The Cobb-Douglas is a constant elasticity function, which means that a 1% change in either of its inputs always results in the same percent change in utility, regardless of the levels of the inputs. To demonstrate, we consider the following logarithmic transformation:

$$\ln(U) = \ln[L^{\alpha} \cdot C^{\beta}]^{\lambda}.$$

It follows that $\ln(U) = \alpha \cdot \ln(L) + \beta \cdot \ln(C)$, and differentiation yields $\beta = \partial \ln(U)/\partial \ln(C)$. Using the approximation $\partial \ln(y)/\partial \ln(x) \approx \%\Delta y/\%\Delta x$, we find that a 1% increase in consumption (C) results in a β% increase in total utility. We call β the *elasticity* of utility with respect to consumption. Similarly, we call α the *elasticity* of utility with respect to leisure.

Budget Constraint

Each household must budget its time between leisure and work. If we introduce $H \equiv$ total hours available for work and leisure, then $W \equiv$ hours of work $= H - L$. Thus, $L(W) = H - W$. C represents consumption expenditure. If we let $w \equiv$ wage rate, then $C(W) = w \cdot W$. Thus, U is an implicit function of W such that $U = U(L(W), C(W))$. The first-order condition provides optimal work effort: $W^* = \beta H/(\alpha + \beta)$.

For example, suppose that there are $H = 126$ hours per week available for work and leisure (18 hours/day). Further suppose that $w = 3$, $\alpha = 0.7$, $\beta = 0.3$, and $\lambda = 3$. These parameters imply that $W^* \approx 38$ hours. The utility function with respect to work effort is shown in Figure 2; we see that utility is maximized at $W^* = 38$.

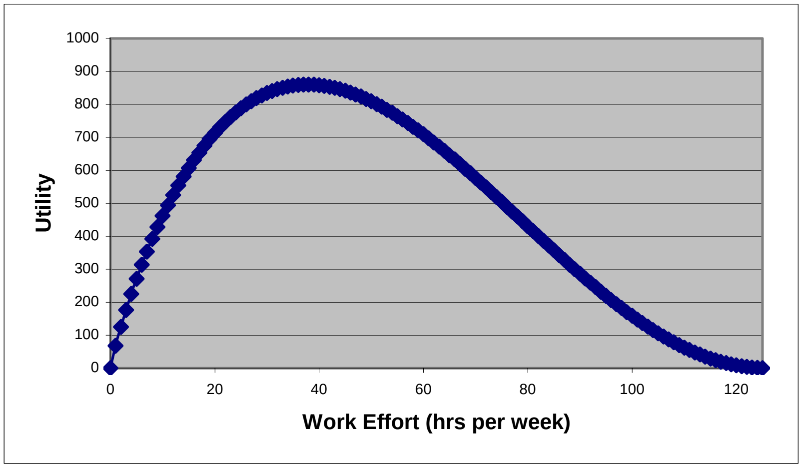

Figure 2. Household utility as a function of work effort.

Once we know the optimal work effort, W^*, as derived above, we can derive the optimal bundle of income and leisure. The optimal household income is $Y^* = w \cdot W^*$. For the one-period model, the household spends all of its income on consumption; thus, the optimal consumption expenditure is $C^* = w \cdot W^* = \$144$, and the optimal leisure is $L^* = 88$ hours. This bundle is represented by a single point in Figure 3, which is identical to Figure 1, but as viewed from above.[14] The optimal point is shown in Figure 3 at the intersection of two lines. This chart illustrates the importance for a household to balance its time between *work* and *play*. All work and no play would put the household on the vertical axis with zero utility. All play and no work would put the household on the horizontal axis with zero utility.

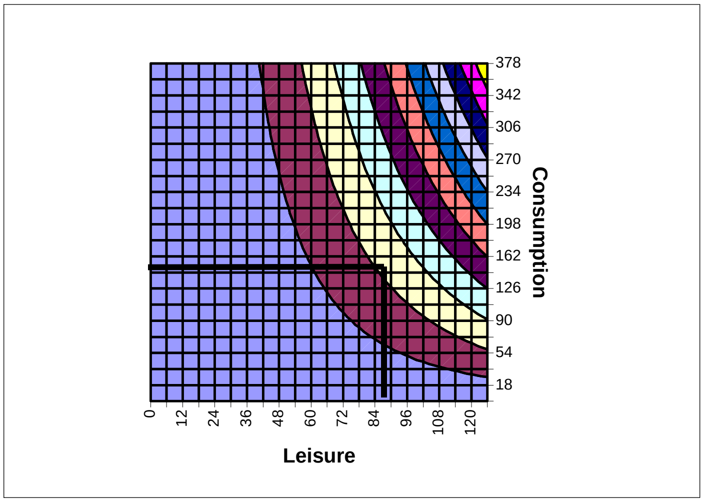

Figure 3. Bird's eye view of household utility function.

[14] The curves in Figure 3 are called preference curves or *indifference* curves because all points on a given curve represent the same level of utility. The slope of an indifference curve at any point is called the marginal rate of substitution; the slope describes how much one good must be increased to make up for the decrease in the other good to maintain the same level of utility (thus making the household *indifferent* toward the trade-off). These conventions can be found in any textbook of economic principles.

Credit Market

The credit market allows borrowers and lenders to exchange liquidity in the form of loans. We shall explore the underlying economic forces that create this market. Consider the following two-period utility function[15]:

$$U = U_1^{\theta 1} \cdot U_2^{\theta 2},$$

where U_t is the utility realized in the t^{th} period. The utility in each period is defined as before: $U_1 = [L_1^{\alpha} \cdot C_1^{\beta}]$, and $U_2 = [L_2^{\alpha} \cdot C_2^{\beta}]$. U is therefore a joint utility for both periods and is plotted in Figure 4.

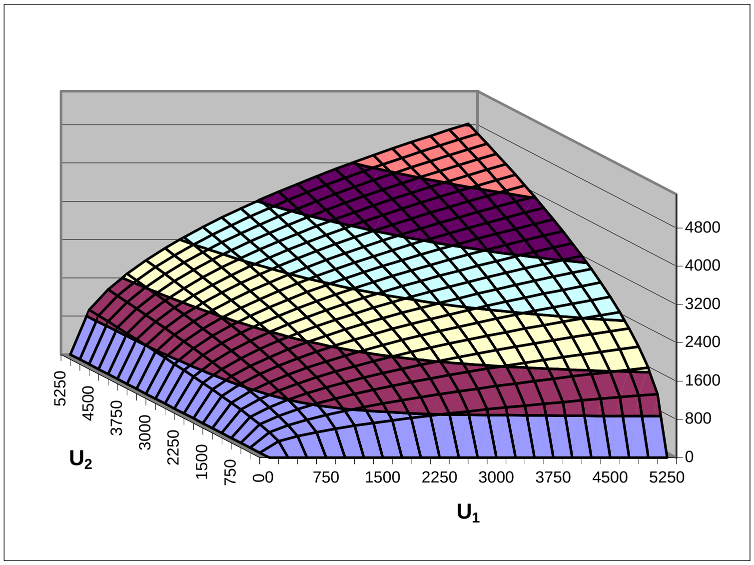

Figure 4. Two-period household utility function.

To explore this relationship, we transform the utility function by the natural logarithm:

$$\ln(U) = \theta_1 \cdot \ln(U_1) + \theta_2 \cdot \ln(U_2).$$

The following partial derivatives provide our interpretation of the temporal coefficients (θ_1 and θ_2):

$$\partial \ln(U)/\partial \ln(U_1) = \theta_1,$$
$$\partial \ln(U)/\partial \ln(U_2) = \theta_2.$$

Using the approximation $\partial \ln(y)/\partial \ln(x) \approx \%\Delta y/\%\Delta x$, we find that a 1% increase in first-period utility results in a θ_1% increase in total utility. Similarly, a 1% increase in second-period

[15] See any microeconomics textbook.

utility results in a θ_2% increase in total utility. To simplify the notation, let u = ln(U) and $u_t = u(C_t) = \ln(U_t) = \ln(U(C_t))$. Thus, the previous equation becomes

$$u = \theta_1 \cdot u_1 + \theta_2 \cdot u_2.$$

If we assume the same parameters from the one-period example for each period in this example, and assume that the household cannot store or borrow consumption across time periods, then θ_1 and θ_2 will not affect the optimal level of consumption and leisure in each period: $C_1^* = C_2^* = \$144$, and $L_1^* = L_2^* = 88$ hours. Therefore, $U_1^* = U_2^*$.

Savings

Now suppose that the household had a mechanism by which it could save or borrow consumption from period 1 to period 2. Let us examine the conditions under which the firm would have an incentive to do so. We derive the first-order condition as follows:

$$du/dC_2 = \partial u/\partial U \cdot \partial U/\partial U_2 \cdot \partial U_2/\partial C_2 + \partial u/\partial U \cdot \partial U/\partial U_1 \cdot \partial U_1/\partial C_1 \cdot dC_1/dC_2 \equiv 0,$$

where

$$\partial u/\partial U = 1/U,$$
$$\partial U/\partial U_t = \theta_t U/U_t,$$
$$\partial U_t/\partial C_t = \beta U_t/C_t.$$

Thus,

$$du/dC_2 = \beta\theta_2/C_2 + (\beta\theta_1/C_1) \cdot dC_1/dC_2.$$

We address the substitution term (dC_1/dC_2) by introducing the two-period budget constraint $M_1 = Y_1 + Y_2/(1 + r)$, which represents the total present value of income from both periods. The term $r \geq 0$ represents the market interest rate, and $Y_2/(1 + r) \leq Y_2$ represents the present (period 1) value of period 2 income when discounted by the market rate. M_1 (budget constraint) provides a borrowing mechanism whereby we can substitute consumption between periods: $M_1 = C_1 + C_2/(1 + r)$. This gives $C_1 = M_1 - C_2/(1 + r)$, so $dC_1/dC_2 = -1/(1 + r) \in [-1, 0)$. Thus, the first-order condition is

$$du/dC_2 = \theta_2/C_2 - (\theta_1/C_1)/(1 + r) \equiv 0.$$

It follows that

$$C_2/C_1 = (1 + r) \cdot \theta_2/\theta_1.$$

Holding w and W_t constant, we now examine three dispositions toward saving and borrowing, as listed in Table 2.

Table 2. Conditions for Saving Between Adjacent Periods

Case	Result	Condition
no substitution	$C_2/C_1 = 1$	$\theta_2/\theta_1 = 1/(1+ r)$
savings	$C_2/C_1 > 1$	$\theta_2/\theta_1 > 1/(1+ r)$
borrowing	$C_2/C_1 < 1$	$\theta_2/\theta_1 < 1/(1+ r)$

Discount Rate

We have determined that a household's disposition toward savings depends upon the relationship between the market interest rate (r) and the household's temporal preferences (θ). This relationship is described by the comparison of two ratios: θ_2/θ_1 against $1/(1 + r)$. We would like to redefine one of the ratios to provide a more intuitive comparison. Since r can be observed and θ cannot, we will redefine θ_2/θ_1 in such a way that we can make a more direct comparison between the market interest rate and each specific household's time preferences. We introduce the internal discount rate $d \equiv (\theta_1 - \theta_2)/\theta_2$, such that $\theta_2/\theta_1 \equiv 1/(1 + d)$. The household discount rate (d) has similar form and interpretation as the market interest rate (r), except that it represents the rate at which the individual household is *willing* to borrow or save.

The internal discount rate (d) can vary across households, whereas the market interest rate (r) is the same for all households. This is because r is determined by the aggregate demand and supply of savings across all households. To explain, $1/(1 + r)$ and $1/(1 + d)$ are prices for substituting between periods. They are analogous to prices in the standard sense. Suppose that P is the market price for televisions, and D is the price that a particular household is willing to pay for a television. If $D > P$, the household will buy the television. If $D < P$, the household will not buy the television. Therefore, some households will buy a television, while others will not; however, all households must make that decision based on the same market price: P. Similarly, if $d > r$, which implies that $1/(1 + d) < 1/(1 + r)$, the household will borrow. If $d < r$, which implies that $1/(1 + d) > 1/(1 + r)$, the household will save.

We assert that $\theta_1 > \theta_2$; that is, households discount the value of *future* consumption relative to *current* consumption. This assertion is supported by the fact that we observe positive interest rates in real-world economies. However, given a market-determined interest rate (r), a household will still only borrow or save based on the relation of its internal discount rate to the market rate. If a particular household has a *relatively* low discount rate, such as $d \in [0, r)$, the household will save[16]. If a particular household has a relatively high discount rate, such as $d > r$, the household will borrow. Note that both of these examples of relative preference are valid even if we assume it is always the case that $\theta_1 > \theta_2$. Therefore, lending and borrowing under a positive interest rate occurs even if all households prefer current to future consumption; that is, even if all households discount the future.

[16] Household savings generally take the form of bank deposits, which support loans to borrowers. Thus, a proper description of credit markets refers to savers as lenders. In this paper, however, we seek only to describe the decisions of individual households, not the structure of the credit market, so we use the term *saver* instead of *lender*.

Financial Planning

To address the household's objectives over a multiple-period time horizon, we extend the original utility function as follows:

$$U = U_1^{\theta l} \cdot U_2^{\theta 2} \cdot \ldots \cdot U_n^{\theta n}.$$

We can generalize the first-order condition for substitution between periods 1 and 2 (from the previous example) to examine substitution between periods 1 and t:

$$du/dC_t = \beta\theta_t/C_t + (\beta\theta_1/C_1) \cdot dC_1/dC_t \equiv 0.$$

The term dC_1/dC_t represents the trade-off in consumption across several time periods. We discount a future value across several periods by applying the discount factor $1/(1 + r)$ for each time period. So, $dC_1/dC_t = 1/(1 + r)^{t-1}$. It follows that

$$C_t/C_1 = (1+r)^{t-1} \cdot \theta_t/\theta_1.$$

As in the two-period model, we can consider the conditions under which the household substitutes between current consumption (in period 1) and future consumption (in period t). These conditions are listed in Table 3.

Table 3. Conditions for Saving Between Any Two Periods

Case	Result	Condition
no substitution	$C_t/C_1 = 1$	$\theta_t/\theta_1 = 1/(1 + r)^{t-1}$
savings	$C_t/C_1 > 1$	$\theta_t/\theta_1 > 1/(1 + r)^{t-1}$
borrowing	$C_t/C_1 < 1$	$\theta_t/\theta_1 < 1/(1 + r)^{t-1}$

In the two-period model, we introduced "d" such that $\theta_2/\theta_1 \equiv 1/(1 + d)$. We now generalize this relationship such that $\theta_t/\theta_1 \equiv 1/(1 + d)^{t-1}$, which implies that households discount between two periods based on the number of intermediate periods. By transforming our utility ($U = \prod U_t^{\theta t}$) into the log-linear form ($u = \sum \theta_t \cdot u_t$) we can divide by θ_1 to obtain

$$u = \sum u_t/(1 + d)^{t-1}.$$

This format assumes that time preference is constant between each adjacent time period. In reality, households' time preferences can change from period to period based on various factors. Such a case of changing time preference would be denoted by a unique discount rate for each period: $u = \sum u_t/(1 + d_t)^{t-1}$, where $d_t \neq d_{t+i}$. Literature pertaining to life-cycle models explores cases of changing time preferences with respect to age-specific factors. However, we find it practical to assume that each household's time preference is constant across time; that is, each household's discount rate is the same for all time periods ($d_t = d_{t+i}$ $\forall$ t,i). We discuss this later.

Substituting the identity $\theta_t/\theta_1 \equiv 1/(1 + d)^{t-1}$ into the first-order condition gives a general relationship between preferred consumption levels across time periods:

$$C_t/C_1 = [(1 + r)/(1 + d)]^{t-1},$$

which allows us to derive some intuitive results. Let $s_t = C_t/C_1$. We have $\partial s_t/\partial r > 0$, which implies that a rise in the market interest rate increases all households' preference for future consumption relative to current consumption, resulting in a general increase in households' propensity to save. We have $\partial s_t/\partial d < 0$, which implies that a rise in an individual household's internal discount rate increases that household's preference for current consumption relative to future consumption.

Budget Constraint (revisited)

For the one-period case, we explored how each household must budget its time between leisure and work. If we assume that work effort conforms to a constant number of hours (W) based on social norms (such as the 8-hour workday or the 40-hour workweek), then leisure is constant. It follows that income (Y) in each period is a function of wage (w): $Y_t = w_t \cdot W$. Thus, the present value of the household's income stream (for T periods) is $B = \sum^T Y_t/(1 + r)^{t-1}$, where r is the *market* interest rate. Assuming that the household has no initial endowment of wealth and that the household can borrow at rate r, it follows that B represents the household's budget constraint for all current and future consumption in present-value terms:

$$\text{Budget Constraint: } \sum^T C_t/(1 + r)^{t-1} \leq \sum^T Y_t/(1 + r)^{t-1}.$$

Any endowments of wealth $A_0 > 0$ in the current period ($t = 0$) are added to the budget constraint in the current period: $B_0 = A_0 + \sum^T Y_t/(1 + r)^{t-1}$. If the household wants to retain a final wealth A_T (discussed below), then the budget is reduced by the discounted value of the final wealth. We have

$$\text{Budget Constraint: } \sum^T C_t/(1 + r)^{t-1} \leq A_0 + \sum^T Y_t/(1 + r)^{t-1} - A_T/(1 + r)^T.$$

This is an underlying constraint for the following discussions of consumption and utility, and it will be explicitly addressed in descriptions of the households' planning algorithm.

Net Worth and the Life Cycle

A household has many factors to consider when planning its future: How many children should it have? How many vacations should it take? What size house should it buy? When should household members retire? Are there any semiretirement options? What are the life expectancies of household members? What is the likelihood of illness or disability? How might disabilities reduce earning potential? We do not want our model to restrict households regarding such choices, nor do we want to explicitly model all of these considerations. We require a model that (1) encapsulates the financial choices associated with these questions and (2) is consistent with the assumption that households will answer these questions differently.

Households also consider the likelihood that their utility will change with age. We get advice such as "Live life while you're young," and "Spend your money while you can enjoy it." These suggestions seem to assume that an individual's utility from consumption will decline with age ($\beta_t > \beta_{t+i}$). This assumption has two implications. First, households will discount consumption in the distant future at a greater rate than consumption in the near future. Second, older households will discount the future at a greater rate than younger households. Of course, the opposite could also be true. Some households might heavily discount current consumption as they work toward their goals for the future. We do not want our model to restrict households regarding such preferences, nor do we want a complicated model of lifetime planning functions with changing time preferences.

We address our model requirements by limiting the household's planning process to periods within a *near-term planning horizon*. The near-term planning horizon is defined simply to be the number of time periods for which the discount rate is constant. The length of this planning horizon can vary across households. The horizon ends with a *target date* (T). The household seeks to achieve a *target new worth* (A_T) by the end of the planning horizon. The target date represents the termination of the current planning horizon. The household's net worth at the end of its current planning phase will be the household's endowment at the beginning of its next planning phase. The household utility under this model is defined as follows:

$$u = \sum^{T} [u_t/(1 + d)^{t-1}] \; + \; u(A_T)/(1 + d)^T.$$

This framework is consistent with many economic interpretations, depending simply on the choice of T and A. We now explore some possible variations and interpretations of this model.

Finite Horizon

It is possible in this framework to assume that $A_T = 0$, which implies that the household plans to consume all of its wealth by date T and have no net wealth at the end. This assumption fits several interpretations. For example, if T is a planned retirement date, and $A_T = 0$, then such a household might expect to be supported solely by its children and social security upon retirement. Alternately, the household might have no plans to retire, and T might represent the anticipated date of death.

Infinite Horizon

Suppose $T = \infty$, which implies that $A = 0$ and $u = \sum^{\infty} u_t/(1 + d)^{t-1}$. This case is studied in the context of various macroeconomic topics, particularly the issue of public debt. We do not intend to employ the case $T = \infty$ in our model, but we present it for completeness and to demonstrate the generalness of our approach for subsequent application.

An infinite planning horizon is particularly simple to employ in a simulation if two conditions are satisfied. Condition 1 is that the function u_t does not change across time periods, which implies that the household's preference for consumption relative to leisure is constant ($\alpha_t = \alpha$ and $\beta_t = \beta \; \forall \; t$). Condition 2 is that the household consumes at a constant level ($C_t = C \; \forall \; t$). These two conditions imply the following [simple] utility function: $u = u_t(C)/d$.

Additive Horizon

The additive horizon, defined when T is finite and $A_T > 0$, provides a simple means to employ a variety of time preferences within the model's agents.

The most obvious interpretation for T is *retirement date*. Under this interpretation, the household plans to earn income through period T and retire with net worth A_T. Net worth could include cash or home equity, or it could include income-generating assets such as stocks, bonds, or real estate.[17] Under this interpretation, households that put money into savings are contributing toward retirement. Such contribution will increase net worth ($\partial A_T/\partial C_t < 0 \ \forall \ t < T$).

A variation of the previous interpretation is a household that is planning to resign from a job at date T and start a business, where the business requires initial start-up capital of A_T.

Of course, T does not have to represent a career change. It could simply represent a financial milestone set by the household for any reason.

Bequeaths

The additive horizon allows us to introduce an infinite horizon component for cases of bequeaths. For example, A_T could represent a trust or an inheritance or some other form of bequeath left by the household as an endowment for beneficiaries. The size of the bequeath will depend on the utility derived by the household out of its concern for the beneficiaries; this utility is denoted by $u(A_T) > 0$. Suppose the endowment takes the form of a trust that will provide a perpetual annuity for some group of beneficiaries. The household knows that an annual payment ($P_{t>T}$) will be made to the beneficiaries, and it derives utility of $u(P_{t>T}) > 0$ from that knowledge. The utility from each payment is discounted, so that the discounted value in period T of $u(P_{t>T})$ will be $u(P_{t>T})/(1 + \delta)$, where δ is the discount rate for payments to beneficiaries. In period T, the household's total present utility from the perpetuity, $u(P_t)/\delta$, will equal the utility of the bequeath itself, A_T. Thus, in time $t < T$, the household can compute its present utility from all future payments to beneficiaries as the discount value of the bequeath itself: $u_{bequeath} = u(A_T)/(1 + d)^{T-t} = u(P_t)/[\delta\cdot(1 + d)^{T-t}]$. Note: This framework allows for $d \neq \delta$, which would imply that the household discounts its own consumption at a different rate than it discounts the consumption of its beneficiaries.

Salient Features

The additive horizon provides a practical and versatile format for handling household planning horizons. First, if empirical data are available, then T and A_T can be generated for the household agents according to some empirically estimated distribution. Second, whether T and A_T are generated from an empirically estimated distribution or some otherwise assumed distribution, the method will allow analysts to study impacts on agent behavior due to variation

[17] We do not need to explicitly define all of the components for A_T. The key here is to assume that the household determines its own target net worth. We need only to define the value of A_T. For the purpose of simulating many households, we can define target net worth as a stochastic variable and generate the value of A_T as the households are created.

in the distribution of T and A_T. Third, T and A_T can be defined by variables in the model, and T and A_T can be continually recalculated by the household based on incoming information or modified by the household as part of the household planning process.

Planning Algorithm

We present a process whereby household agents behave according to an evolutionary learning algorithm. In each planning phase, the household estimates its current fitness (derived in part from financial variables), determines its current state of being, and specifies the values for two choice variables: consumption and savings. The process could be extended to include investment (a risky form of savings) as a third choice variable, but we shall ignore investment in this discussion.

Fitness

Fitness is an estimate of expected utility. *Utility* is defined as the discounted value of the household's planned consumption stream from now (t = 0) through time T:

$$\text{Utility} \equiv \sum^{T} u(C_t)/(1 + d)^{t-1} + u(A_T)/(1 + d)^{T}.$$

This function is a Von Neumann–Morgenstern utility function, and is therefore valid even if the households are uncertain regarding future consumption levels ($C_{t>1}$), perhaps due to a dependence on uncertain prices, or uncertain regarding future time preferences ($d_{t>1}$) or consumption preferences ($\beta_{t>1}$), perhaps due to uncertain health[18]. We introduce the notation E[x] to denote the *expected value of x* based upon some estimation process. Let u_t^e denote $E[u_t]$ and C_t^e denote $E[C_t]$, so that

$$\text{Expected utility} \equiv \sum^{T} u^e(C_t^e)/(1 + d)^{t-1} + u^e(A_T^e)/(1 + d)^{T}.$$

The introduction of time-dependent and state-dependent uncertainties, such as those mentioned above, restricts the household's ability to forecast future utility, and therefore restricts its ability to estimate expected utility for a set of choice variables.

State Variables

We address the uncertainties described above by introducing a set of state variables (Ψ) to represent all exogenous variables in the utility function.[19]

---

[18] In either or both cases, the utility is a monotonic *linear* transformation of the expected utilities in each time period, $E[u_t] = \int u_t/C_t \, dC_t$, and satisfies the expected utility property.

[19] In this model, C_t^e, A_T^e, and T are examples of possible choice variables because they can be set by the household during the planning process to increase utility. In contrast, d_t (discount rate), β_t (consumption elasticity), and w_t (wage) are examples of state variables because they are determined outside of the household's planning process; they are either imposed by the simulation or set by other agents.

Savings

A household must allocate its income between consumption and savings. Let U_0 denote expected utility in the current time period. The household seeks to maximize

$$U_0 \equiv U_0(C_0, S_0 \mid \Psi_0, \{\Psi_t\}, B),$$

where C_0 is the current consumption expenditure, S_0 is the current allocation to savings, Ψ_0 is the current state, $\{\Psi_t\}$ is the set of future states, and B is the expected all-period budget constraint.

Savings is defined with respect to current income (Y_0) and current net worth (A_0) as follows: $S_0 = Y_0 - C_0$, which implies $dS_0/dC_0 = dC_0/dS_0 = -1$. If we assume that all states are fixed, then

$$dU_0/dS_0 |_\Psi = \partial U_0/\partial S_0 + \partial U_0/\partial C_0 \cdot dC_0/dS_0 = \partial U_0/\partial S_0 - \partial U_0/\partial C_0.$$

The term $\partial U_0/\partial C_0 > 0$ is the partial utility of *current* consumption. The term $\partial U_0/\partial S_0 > 0$ is the partial utility of *future* consumption (via savings)[20]. The net utility from additional savings could be positive or negative depending on the relative utility from current versus future consumption.

State Transition

We allow for the possibility that a household's *current* choices might influence its future states: $\partial\{\Psi_t\}/\partial S_0 \neq 0$. Let $\Psi \equiv \{\Psi_t\}$. We have

$$dU_0/dS_0 = dU_0/dS_0 |_\Psi + \partial U_0/\partial\Psi \cdot \partial\Psi/\partial S_0.$$

Now we allow for multiple state variables ($\Psi_t = \{\psi_{it}\}$) to obtain

$$dU_0/dS_0 = dU_0/dS_0 |_\Psi + \textstyle\sum_i \partial U_0/\partial\psi_i \cdot \partial\psi_i/\partial S_0.$$

Example of an Assessment Step

In any planning phase, given the current state, a household must try to determine two results that might occur from further savings. First, will the increased utility from additional future consumption outweigh the decreased utility from foregone current consumption? Second, will further savings move the household into a better or worse state for future decisions?

We will not define an optimization problem whereby households try to derive an optimal amount of savings analytically, using mathematical calculations. Instead, the households will learn how to respond in specific states. In Table 4, we define three *direct state* variables corresponding to utility.

[20] We have $\partial U_0/\partial S_0 = \sum^T \partial C_t^e/\partial S_0 + \partial A_T^e/\partial S_0 > 0$. The magnitude of each of the T+1 partial derivatives on the right-hand side of this equation depends on the household allocation of savings and all future states, but we know that each term is ≥ 0.

Table 4. Definition of State Variables

ψ_1	= 1 = 0	if consumption is increasing: $dC_t/dt > 0$ otherwise
ψ_2	= 1 = 0	if assets are increasing: $dA_t/dt > 0$ otherwise
ψ_3	= 1 = 0	if assets exceed target: $A_0 > A_T$ otherwise

In this case, we have n = 3 state variables. For any such collection of *indicator* (true or false) state variables, there are $N = 2^n$ states. In this case, we have $N = 2^3 = 8$ states for period t, where the set of possible *direct states* in period t is denoted by $\Psi_t = \Psi\{\psi_{1t}, \psi_{2t}, \psi_{3t}\}$.

We also want households to respond with regard to state transitions. We introduce a new set of *Markov* states: $\Phi_t = \Phi\{\Psi_t, \Psi_{t-1}\}$. There are $M_t = N_t \cdot N_{t-1}$ possible Markov states (in our model, N and M are constant across time). For our example, we have the following N = 8 direct states:

Ψ	ψ_1	ψ_2	ψ_3
1	1	1	1
2	1	1	0
3	1	0	1
4	1	0	0
5	0	1	1
6	0	1	0
7	0	0	1
8	0	0	0

And we have the following M = 64 Markov states:

Φ_t	Ψ_t	Ψ_{t-1}
1	1	1
2	1	2
3	1	3
4	1	4
5	1	5
6	1	6
7	1	7
8	1	8
9	2	1
10	2	2
11	2	3
…	…	…
62	8	6
63	8	7
64	8	8

Example of a Decision Step

A household must choose a level of savings. We partition this choice into two components: (1) an integer component for *disposition* (σ_D) and (2) a floating-point component for *magnitude* (σ_M). Disposition denotes the direction of the action taken, and magnitude denotes the extent of the action taken. For example, the disposition of a household with respect to savings can fall into one of three discrete categories: (1) the household might choose to save some of its income; (2) the household might choose to spend more than its income by borrowing extra money; or (3) the household might spend the exact amount of its income, neither saving nor borrowing. These three options are represented in the following chart by three possible values of σ_D:

σ_D	$= 1$	save: $S_0 = Y_0 - C_0 > 0$
	$= -1$	borrow: $S_0 = Y_0 - C_0 < 0$
	$= 0$	neither

Once a household has decided its *disposition* toward saving, it must decide the corresponding *magnitude* for this action. For example, if a household decides to be a saver, will it be a big saver, a moderate saver, or a minor saver? Magnitude denotes the level of savings as a share of income: $\sigma_M = S_0/Y_0 \in [0, 1]$. We partition the savings decision for the following reasons. In principle, it seems reasonable for a household to first ask, “Should I save or borrow?” and then ask, “OK, how much?” However, σ_D has a much more profound interpretation. It represents the household’s disposition toward savings given a particular state of being. It allows us to discretely classify the household as a *saver* or a *borrower* for a given state, which is an important distinction in credit markets.

Disposition: In practice, the disposition component defines a discrete set of options from which a household can choose. This is an important feature for the sake of the decision algorithm, wherein we want the household to learn how to respond in certain circumstances.

For a given Markov state, say $\Phi_0 = 5$, a household has three options:

Φ_0	Ψ_0	Ψ_{-1}	σ_D
5	1	5	1
5	1	5	0
5	1	5	–1

This policy table contains quite a bit of information. It shows that the household was previously in direct state 5, which implies the previous period had consumption decreasing ($\psi_1 = 0$), savings increasing ($\psi_2 = 1$), and assets that exceeded the long-term asset goal ($\psi_3 = 1$). The chart shows that the household is currently in direct state 1, in which consumption is now increasing ($\psi_1 = 1$).

To understand how the household selects from the three options, we introduce a strength parameter: $\omega \in [0, 1]$. This represents the strength of each option in the current state, which determines the probability of selecting each option. For this example, we would have initiated all options at the start of the simulation to have equal strength ($\omega = 0.33$). However, each time an option is selected throughout the simulation, ω is increased or decreased depending on whether the decision resulted in a higher fitness for the household. Suppose our current options have the following strengths:

Φ_0	Ψ_0	Ψ_{-1}	σ_D	ω	σ_M
5	1	5	1	0.3	0.2
5	1	5	0	0.5	
5	1	5	–1	0.2	0.3

Choice and Magnitude: Once the household decides whether to be a saver, a borrower, or neither, it must decide the extent of its disposition. For example, "I will be a big borrower," or "I will borrow much." The three rows in the chart above denote three prospects: (1) a 30% chance of saving 20% of income, (2) a 50% chance of spending all income, and (3) a 20% chance of borrowing 30% of income. The rightmost column, σ_M, contains the percentage of savings/borrowing in which the household will engage, once the disposition, σ_D has been selected.

Example of an Evaluation Step

Suppose that the algorithm selects the first option (save 20% of income). The household will then set its expenditure variable accordingly and proceed through a series of execution steps in which the household exchanges and adjusts its cash balance. When the household gets to the next decision phase, it will determine whether fitness has increased or decreased.

Suppose that fitness has increased. In this case, the household concludes that it made the correct choice for the given situation and increases the parameter $\omega_{(\Phi=5,\ \sigma\ \sigma D=1)}$ by some proportion. This step will increase the strength of that option (savings) and thereby increase the probability of selecting the same option the next time the household finds itself in state $\Phi = 5$. If fitness had decreased, then the household would have decreased the strength of the *savings* option.

Continuing with the supposition that fitness has increased, we want the household to consider whether it saved the proper proportion of income. Based on well-founded economic principle and our theoretical framework, the household experiences diminishing returns from increased consumption in any time period. This implies that increased savings will only benefit the household to a point, but fitness will eventually begin to decline as current consumption approaches zero. We incorporate this principle by allowing the firm to adjust σ_M whenever it adjusts ω. Thus, once the household determines that savings are beneficial, it increases both ω and σ_M. Suppose the household repeatedly finds itself in state $\Phi = 5$, and increasingly selects the first option (savings) until σ_M approaches *unity* and fitness starts to decline upon evaluation. The household will then begin to reduce the amount of savings whenever the savings option is chosen. By adjusting the magnitude parameter, we expect the household to converge to an optimal proportion of income devoted to savings.[21]

[21] Convergence of σ_M is only for those cases when the household selects the correct option. In our discussion, the economic structure causes the proportion of savings to converge to $\sigma_M < 1$. Unfortunately, the algorithm described here will also cause the strength to converge to $\omega < 1$ even if it should not; that is, even if the *savings* option always improves fitness and other options always reduce fitness. We will correct this problem in the future.

Remarks

The learning algorithm proposed above describes a general approach, which we will likely modify and extend upon implementation in an actual simulation. Nevertheless, we expect that the underlying economic framework and evolutionary learning method presented here will provide a sound and practical starting point for addressing the role of household financial planning in economic simulations.

References

Ando, Albert, and Modigliani, Franco. “The ‘Life Cycle’ Hypothesis of Saving: Aggregate Implications and Tests.” *American Economic Review* 53 (March 1963): 55–84.

Arai, Mahmood. “Wages, Profits, and Capital Intensity: Evidence from Matched Worker-Firm Data.” *Journal of Labor Economics* 21 (July 2003): 593–618.

Barro, Robert J. “The Ricardian Approach to Budget Deficits.” *Journal of Economic Perspectives* 3 (Spring 1989): 37–54.

_____. *Macroeconomics, 3rd ed*. New York: John Wiley & Sons, 1990.

Becker, Gary S., and Mulligan, Casey B. “The Endogenous Determination of Time Preference.” *Quarterly Journal of Economics* 112 (August 1997): 729–58.

Caplin, Andrew, and Leahy, John. “Psychological Expected Utility Theory and Anticipatory Feelings.” *Quarterly Journal of Economics* 116 (February 2001): 55–79.

Christensen, Laurits R.; Jorgenson, Dale W.; and Lau, Lawrence J. “Transcendental Logarithmic Utility Function.” *American Economic Review* 65 (June 1975): 367–83.

Clark, Andrew E. “Unemployment As a Social Norm: Psychological Evidence from Panel Data.” *Journal of Labor Economics* 21 (April 2003): 323–52.

Dreze, Jacques, and Modigliani, Franco. “Consumption Decisions under Uncertainty.” *Journal of Economic Theory* 5 (1972): 308–35.

Dynan, Karen E. “How Prudent Are Consumers?” *Journal of Political Economy* 101 (December 1993): 1104–13.

Dynan, Karen E.; Skinner, Jonathan; and Zeldes, Stephen P. “Do the Rich Save More?” *Journal of Political Economy* 112 (April 2004): 397–444.

Feather, Peter M., and Shaw, W. Douglass. “The Demand for Leisure Time in the Presence of Constrained Work Hours.” *Economic Inquiry* 38 (October 2000): 651–61.

Hubbard, R. Glenn; Skinner, Jonathan; and Zeldes, Stephen P. “Precautionary Saving and Social Insurance.” *Journal of Political Economy* 103 (April 1995): 360–99.

Silverberg, Eugene. *The Structure of Economics: A Mathematical Analysis, 2nd ed*. New York: McGraw-Hill Publishing Company, 1990.

Tranæs, Torben. "Raiding Opportunities and Unemployment." *Journal of Labor Economics* 19 (October 2001): 773–98.

Von Neumann, J., and Morgenstern, O. *Theory of Games and Economic Behavior*. Princeton: Princeton University Press, 1944.

Price Formation in the Stock Market

We model the stock market as a continuous double auction conducted by a stock exchange agent that takes limit orders from household agents. Each order includes a buy/sell indicator, the number of shares to be traded, and the limit price for the order.[22] Households seeking to participate in the market will submit limit orders based on recently reported stock prices. The exchange agent simply clears the market of open orders. We are not concerned with the precise rule by which the exchange agent clears the market, except that it satisfies some reasonable requirements (see Farmer 2001 for examples).

In our model, households are investors, not arbitragers; we do not seek to model trading strategies. Households treat stocks as a high-risk, high-yield alternative to savings. Therefore, we are not concerned how each household selects a limit price, only that the limit prices satisfy some aggregate properties (Ilija and Farmer 2002). The deviation between market price and limit price will be random and will follow a power law. We are not concerned in this model whether the variance (of deviations of limit prices from market price) is constant across all households or varies across individual households or arbitrary groupings of households.

References

Domowitz, Ian, and Wang, Jianxin. “Auctions As Algorithms.” *Journal of Economic Dynamics and Control* 18 (1994): 29–60.

Farmer, J. Doyne. “Toward Agent-Based Models for Investment.” In *Developments in Quantitative Investment Models*. AIMR, 2001.

Farmer, J. Doyne, and Lillo, Fabizio. “On the Origin of Power Law Tails in Price Fluctuation.” *Quantitative Finance* 314 (2004): C7–11.

Farmer, J. D.; Patelli, Paolo; and Zovko, Ilija. “The Predictive Power of Zero Intelligence Models in Financial Markets.” Los Alamos National Laboratory Condensed Matter Archive, 2003.

Gomme, Paul; Kydland, Finn E.; and Rupert, Peter. “Home Production Meets Time to Build.” *Journal of Political Economy* 109 (October 2001): 1115–31.

Harris, Lawrence, and Hasbrouck, Joel. “Market vs. Limit Orders: The SuperDOT Evidence on Order Submission Strategy.” *Journal of Financial and Quantitative Analysis* 31 (1996): 213–31.

[22] The *limit price* tells the exchange the least-best price at which the buyer or seller wants to trade. For a buy-stock order, the limit price is the maximum price the buyer wants to pay for the stock; the buy order will not be executed unless and until the market price falls below the limit price. For a sell-stock order, the limit price is the minimum price the seller wants for selling the stock; the sell order will not be executed unless and until the market price rises above the limit price.

Huffman, Gregory W. “A Dynamic Equilibrium Model of Asset Prices and Transaction Volume.” *Journal of Political Economy* 95 (February 1987): 138–159.

Lo, Andrew W.; MacKinlay, A. Craig; and Zhang, June. “Econometric Models of Limit-Order Executions.” *Journal of Financial Economics* 65 (2002): 31–71.

Mendelson, Haim. “Market Behavior in a Clearing House.” *Econometrica* 50 (November 1982): 1505–24.

Pástor, Lubos, and Stambaugh, Robert F. “Liquidity Risk and Expected Stock Returns.” *Journal of Political Economy* 111 (June 2003): 642–685.

Reynolds, Kate. “Going, Going, Gone! A Survey of Auction Types.” *Agorics*. 1996. Available at http://www.agorics.com/Library/auctions.html (accessed 17 July 2004).

Zovko, Ilija, and Farmer, J. Doyne. “The Power of Patience: A Behavioral Regularity in Limit-Order Placement.” *Quantitative Finance* 2 (2002): 387–392.

Distribution

1	0161	Patent & Licensing Office	11500
1	9018	Central Technical Files	08945-1
2	0899	Technical Library	09616
1	0321	W. J. Camp	09200
1	1110	D. Womble	09210
10	0318	J. E. Nelson	09209
64	0318	S. G. Wagner	09209
3	0196	J. A. Sprigg	09216
3	0318	C. R. Jorgensen	09216
3	1109	R. J. Pryor	09216
1	1138	S. M. Deland	06222
1	1138	L. A. Snyder	06222
1	1138	M. A. Ehlen	06221
1	1138	S. M. Rinaldi	06221
1	0839	N. H. Hayden	16000
1	1209	J. M. Chavez	05920
1		Steve Ashby – LLNL	
1		David Brown – LLNL	
1		Doug Speck – DHS	
1		John Cummings – DHS	